RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2016
FIRST YEAR [BATCH 2015-18]

Date : 18/05/2016 **PHYSICS (Honours)**
Time : 11 am – 3 pm **Paper : II** Full Marks : 100

[Use a separate Answer Book for each group]

Group – A

Answer any four questions from question no. 1 to 7 : [4×10]

1. a) Write down the complex form of Fourier series. Obtain the complex form of the Fourier series of the function $f(x)=\begin{cases}0 & -\pi<x<0\\ x & 0<x<\pi\end{cases}$. [1+5]

   b) Calculate the Fourier transform g(k) of the following function: $f(x)=\begin{cases}1 & for\ |x|<a\\ 0 & for\ |x|>a\end{cases}$ [2+1+1]

2. a) Show that the following function can be expressed as delta function for $n\to\infty$: $\delta n(x)=\dfrac{Sin\,nx}{\pi x}$ [4]

   b) Find $\int_{-1}^{1} 9x^3\delta(3x+1)dx$ [1]

   c) Write down the Laplace's equation in spherical polar co-ordinate. Separate out Θ – part and show it leads to Legendre equation. [1+4]

3. a) Determine the potential within a rectangular box having dimension (a, b, c) along (x, y, z) directions with the following boundary conditions: [5]
   i) $\phi(x,y,z)=0$, for $x = 0$, $y = 0$, $z = 0$
   ii) $\phi(x,y,z)=0$, for $x = a$ and $y = b$
   iii) $\phi(x,y,z)=f(x,y)$, for $z = c$

   b) Solve the equation $\dfrac{\partial U}{\partial t}=\dfrac{\partial^2 U}{\partial X^2}$, $t > 0$, $0 < x < L$, using the method of separation of variables, subject to the boundary condition $U(0,t)=0=U(L,t)$ for all t. [5]

4. a) A particle of mass m moves under the action of a central force $f(r)\hat{r}$. Find the differential equation of motion of the particle. Hence show that the trajectory of the particle moving under an inverse square force field would be a conic section. [4+2]

   b) The orbit equation of a particle in central force field is given by $r=a(1+\cos\theta)$. Show that the force field is attractive and $f(r)=-\dfrac{k}{r^4}$ where $k > 0$. [4]

5. a) State Kepler's laws of planetary motion. [1]

   b) Show that these laws can be derived from the equation of motion of a planet moving under an attractive inverse-square law of force. [6]

   c) An Earth Satellite moves in an elliptic orbit with a period T_1 eccentricity e, and semi major axis a. Show that the maximum radial velocity of the satellite is, $2\pi ae/(T\sqrt{1-e^2})$. [3]

6. a) An observer stationed at a point which is fixed relative to an xyz coordinate system with origin O observes a vector $\vec{A}=A_1\hat{i}+A_2\hat{j}+A_3\hat{k}$ and calculate its time derivative to be $\dfrac{d\vec{A}}{dt}=\dfrac{dA_1}{dt}\hat{i}+\dfrac{dA_2}{dt}\hat{j}+\dfrac{dA_3}{dt}\hat{k}$. Later he finds that he and his coordinate system are actually

rotating with respect to an *XYZ* coordinate system taken as fixed in space and having origin also at *O*. Show that the time derivative of $\vec{A}$ for an observer who is fixed relative to the *XYZ* coordinate system is given by, $\left.\frac{d\vec{A}}{dt}\right|_F = \left.\frac{d\vec{A}}{dt}\right|_M + \vec{W}\times\vec{A}$,

where the subscript *F* and *M* devotes the fixed and moving system respectively. [5]

b) A object is thrown downward with initial speed v_0. Prove that after time *t* the object is deflected east of the vertical by the amount $wv_0t^2\sin\lambda + \frac{1}{3}wgt^3\sin\lambda$, where *w* is the angular velocity of Earth, *g* is the effective acceleration due to gravity and λ is the colatitude of the position. [5]

[Hint: Equation of motion of a particle near the Earth's surface is given by, $\frac{d^2\vec{r}}{dt^2} = \vec{g} - 2(\vec{w}\times\vec{v})$]

7. a) Show that the moment of inertia of a rigid body rotating about the axis passing through the origin and having direction cosine (*l*, *m*, *n*) is given by

$$I_{l,m,n} = l^2 I_{xx} + m^2 I_{yy} + n^2 I_{zz} + 2lmI_{xy} + 2mnI_{yz} + 2nlI_{zx}$$

How is the expression modified for the principal axis system? [4]

b) For a thin uniform square plate of side *a* and mass *m*, what will be the principal axes of inertia? [2]

c) Find the moment of inertia of a homogeneous cube of side *a* and mass *M* about any axis passing through its centre. [4]

Group – B

Answer any two questions from question nos. 8 to 11 : [2×10]

8. a) Write down Laplace's Equation and Poisson's Equation for gravitational potential. [2+2]

b) Find out the gravitational potential at the centre of curvature of an uniform rod bent into the form of a circular arc. [2]

c) Write a short note on Binary Star. If the binary stars have masses m_1 and m_2 and are at a distance *r* apart find out the period of rotation, compare their angular momenta and kinetic energies. [4]

9. a) Find out the relation between bulk modulus, rigidity modulus, Young's modulus and Poisson's ratio. [6]

b) Find out the limiting values (maximum & minimum) of Poisson's ratio. [1]

c) Young's modulus and rigidity moduli of a substance are $7\text{x}10^{11}$ dynes/cm^2 and $3\text{x}10^{11}$ dynes/cm^2 respectively. Calculate the bulk modulus and Poisson's ratio of the substance. [3]

10. a) Define surface tension, surface energy and angle of contact. [3]

b) Find out the expression of excess pressure on a curved surface (membrane) of Principal radii r_1 and r_2 and uniform surface tension *s*. [4]

c) Find the value of excess pressure in case of a spherical soap bubble, a cylindrical soap bubble and in a liquid drop or an air bubble in a liquid. [3]

11. a) What is viscosity & coefficient of viscosity. Define critical velocity. [1+1]

b) A liquid is flowing in streamline through a horizontal capillary tube. Find out the expression for the volume of liquid flowing through it per unit time. Include the energy correction in this expression. [3+1]

c) Derive Bernoulli's theorem from Euler's equation of motion. [4]

Group – C

Answer any four questions from question nos. 12 to 18 : [4×10]

12. a) Derive the differential equation for wave motion in one dimension, and obtain a plane wave solution. [4]

b) Prove that the velocity of sound waves in a long solid medium is $\sqrt{\frac{Y}{\rho}}$, where Y is the Young's modulus and ρ is the density of the medium. State clearly the assumptions mode. [5+1]

13. a) What do you mean by phase and group velocity of a wave? Derive a relation between the two velocities. [2+3]

b) For gravity waves in a liquid the phase velocity C depends on the wave length λ according to the formula $C = A\sqrt{\lambda}$, A being a constant. Show that the group velocity is half the phase velocity. [1]

c) A source of sound is approaching an observer with velocity u, and the observer is also approaching the source with velocity v. If f be the emitted frequency, calculate the frequency measured by the observer, assuming the wind to be stationary. [4]

14. a) The two ends of a stretched string are fixed at x = 0 and x= ℓ. It is struck at x = a at t = 0. Write down the initial conditions and analyse the subsequent motion of the string. [6]

b) Find the amplitudes of the fundamental tone at $x = \frac{\ell}{2}$ and $x = \frac{\ell}{4}$ when struck at $a = \frac{\ell}{3}$. [2]

c) State the differences between the vibration modes of a struck and plucked string. [2]

15. a) Find out an expression for the potential energy of a vibrating stretched string of mass M, length L, fixed at both ends. [Assume the expression for the displacement] [3]

b) A traffic police is blowing a whistle of frequency 200 Hz. If the sound is reflected from an oncoming bus moving with speed 30km/hr. What would be the number of beats as heard by a stationary observer? [2]

c) Explain Fresnel's half-period zones. Find the expression of the radius of mth boundary of Fresnel zones and show that the area of individual zones are approximately same. [5]

16. a) Derive an expression for the transmittivity (T) of a plane parallel film of thickness h and refractive index n_2 surrounded by medium of refractive index n_1 due to multiple reflections at the two interfaces of the film. Plot the transmittivity (T) as function of phase difference between two successive waves from the film (δ). [5+1]

b) How can you find the wavelength of monochromatic light with Michelson interferometer? [2]

c) In an experiment with Michelson interferometer, the distance through which the mirror is moved between two consecutive positions of disappearance of fringes is 0·2945 mm. If the mean wavelength of the two components of the D lines of sodium light is 589·3 nm, find the difference between their wavelengths in nm. [2]

17. a) Show that in Fresnel's biprism setup if d_1 and d_2 are the distances between the virtual images of the slit for two different position of a convex lens placed in between the biprism and the screen then the distance between the virtual images can be determined using the relation $d = \sqrt{d_1 d_2}$.

Explain the theory for determination of wavelength of monochromatic light using Fresnels' biprism. [4+2]

b) Explain coherence length and coherence time of light waves. Find the bandwidth ($\Delta\lambda$ in nm) of Mercury arc light with mean wavelength 550 nm, and coherence length of 0·3 mm. [2+2]

18. a) Distinguish between Fraunhofer and Fresnel diffraction. Derive the expression for the intensity distribution of Fraunhofer diffraction pattern due to a single slit. [2+4]

b) A single-slit Franhofer diffraction pattern is formed using white light at normal incidence. For what wavelength of light does the third maximum coincide with the fourth minimum for light with wavelength $4\cdot0\times10^{-5}$ cm? [3]

c) Explain Rayleigh criterion of resolution. [1]

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